

Repulsive and restoring Casimir forces with left-handed materials

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Abstract

We investigate repulsive Casimir force between slabs containing left-handed materials with controllable electromagnetic properties. The sign of Casimir force is determined by the electric and magnetic properties of the materials, and it is shown that the formation of the repulsive force is related to the wave impedances of two slabs. The sign change of the Casimir force as a function of the distance is studied. Special emphasis is put on the restoring Casimir force which may be found to exist between perfectly conducting material and metamaterial slabs. This restoring force is a natural power for the system oscillation in vacuum and also can be used for system stabilization.

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It is well known that the change of the zero point energy of quantized electromagnetic field in the presence of the boundary surface gives rise to forces on macroscopic bodies. The existence of the forces called Casimir effect has attracted considerable attention over decades [1]. Various calculation techniques have been developed where systems of different geometries of the boundary were considered, and corrections to the idealized system were made in any practical situation. In recent years, the force can be measured in experiment with the development of microelectromechanical and nanoelectromechanical systems (MEMS and NEMS) and nanotechnology. Since the attractive interaction of a pair of neutral perfectly conducting parallel plates placed in the vacuum is theoretically proposed in 1948 by Casimir [2], the forces have been known to be attractive in most cases. The attractive Casimir forces could lead to stiction problem in MEMS and NEMS, and therefore, the repulsive forces may avoid that limits and are of possible practical significance [3, 4].

Recently, artificial composite metamaterials with controllable electromagnetic properties, namely left-handed material (LHM) [5, 6, 7, 8] and single-negative (SNG) material [9, 10, 11], were fabricated experimentally. The LHM has simultaneously negative permittivity and permeability over a band of frequencies and thereby the refractive index is negative, while SNG material has only one negative material parameter within a certain frequency range, including the ϵ -negative (ENG) media with $\epsilon < 0$ (but $\mu > 0$) and the μ -negative (MNG) media with $\mu < 0$ (but $\epsilon > 0$). They may possess noticable magnetic properties. The LHM was originally predicted in the theoretical point of view by Veselago [12] where an alternative solution to Maxwell's equations is provided. The electromagnetic wave propagating in an LHM has the wave vector opposite to the direction of energy flow. The directions of the electric field, the magnetic field and the wave vector form a left-handed system. Such unusual phenomena as reverse Doppler effect, Cherenkov radiation, anomalous refraction and reversal of radiation pressure, are expected in the LHM. More possible applications, such as perfect lens [13], indirect quantum interference [14], have been proposed. Recently the Casimir effect between LHM slabs has been investigated [15]. Perfect lens is introduced in the planar geometry to obtain repulsive Casimir force [16]. SNG materials have also exhibited special features. For example, unique transmission properties have been found in SNG multilayers [17, 18].

In the present work, we analyze the repulsive Casimir effect between two parallel slabs and the sign change of the force with respect to the slab separation. The metamaterials are taken

into consideration. It is known that the repulsive Casimir force is possible between two bodies that possess an asymmetric nature [19]. The difference of the electromagnetic properties between two materials plays an important role in forming the repulsive force. Starting from the relationship between the wave impedances of two materials and the condition for the formation of the repulsive Casimir force, we study the different configurations of parallel slabs, including the ordinary dielectric materials, the metamaterials, and even the perfectly conducting materials. One may adjust the characteristic frequencies of the metamaterials in order that the repulsive forces can be found. In particular, we focus on a special case of sign change of the force: the repulsive force becomes attractive with the increasing slab separation. This kind of restoring force, which may exists between a perfectly conducting slab and a metamaterial slab, could make the slabs oscillating, or stabilize the two slabs at a fixed distance.

Let us consider the configuration with two parallel infinite slabs, A and B, each of thickness d separated by a distance a , in free space. Based on the stress tensor method using the properties of the macroscopic field operators [20], we can calculate the Casimir force, which is eventually expressed as

$$F_C = \frac{\hbar}{2\pi^2} \int_0^\infty d\xi \int_0^\infty k dk \sqrt{\frac{\xi^2}{c^2} + k^2} \sum_{N=\text{TE, TM}} \frac{r_N^A(\xi, k) r_N^B(\xi, k) e^{-2a\sqrt{\xi^2/c^2 + k^2}}}{1 - r_N^A(\xi, k) r_N^B(\xi, k) e^{-2a\sqrt{\xi^2/c^2 + k^2}}} \quad (1)$$

where $r_N^{A(B)}$ is the slab reflection coefficient for TE- and TM-polarized waves. It is seen from Eq. (1) that the integrand can be negative only if the reflection coefficients of the two slabs, $r_N^A(\xi, k)$ and $r_N^B(\xi, k)$, have different signs, which contributes to the formation of the repulsive force. Therefore, the repulsive Casimir effect is to be expected when the two parallel slabs have different electromagnetic properties, and that is to say, the force between two identical slabs must be attractive, just as has been recently proved in Ref. [19]. The van der Waals force between a magnetically polarizable particle and a electrically polarizable particle is repulsive [21], and similarly, repulsive Casimir force is found between a perfectly conducting plate and an infinitely permeable plate [22], which can be easily calculated from Eq. (1) by considering $\epsilon_A \rightarrow \infty$ and $\mu_B \rightarrow \infty$: $F_C = -7\hbar c\pi^2/1920a^4 = -\frac{7}{8}F_0$, where $F_0 = \hbar c\pi^2/240a^4$ is the well-known formula for the attractive Casimir force between two perfectly conducting plates. A further fact is known that the forces are possibly repulsive between two molecules when the medium they immersed in has the intermediate properties between the properties

of two polarizable molecules [23]. We thus can apply an analogous analysis for the system of two parallel slabs in vacuum: The repulsive behavior may possibly appear when the wave impedances $Z_{A(B)} = \sqrt{\mu_{A(B)}/\epsilon_{A(B)}}$ of two slabs, which are used to demonstrate the difference of electric and magnetic properties between two slabs, are smaller and larger than the impedance of vacuum, respectively.

We consider the effect for the semi-infinitely thick slabs, which is an approximate model following the condition of $a \ll d$, and the slab reflection coefficients are simplified to the single interface reflection coefficients. We model the dispersive material constants by the single-resonance Drude-Lorentz type

$$\{\epsilon, \mu\} = 1 + \frac{\omega_{P\nu}^2}{\omega_{T\nu}^2 - \omega^2 - i\gamma_\nu\omega}, \quad (2)$$

($\nu = e, m$, which refer to ϵ and μ , respectively) where $\omega_{P\nu}$ is the plasma frequency, $\omega_{T\nu}$ is the resonant frequency and γ_ν is the damping frequency. In the following, ω_0 denotes a unit of the frequency with which the characteristic frequencies are scaled, and $\lambda_0 = 2\pi c/\omega_0$ is the corresponding wavelength in the vacuum.

For the dispersive ordinary dielectric material slabs with trivial constant permeability $\mu = 1$, most values of the impedances of two slabs over the whole frequency range are no larger than unity, i.e., the impedance of vacuum. Thus the integrand mainly contributes to the attractive forces, and it is hard to obtain the repulsive Casimir forces. However, the trend towards formation of the repulsive forces may be clearly seen from the cases of the material slabs of constant impedances. The dependence of the relative force $F_r = F_C/F_0$ between two idealized, non-dispersive ordinary dielectric slabs on the permittivities is shown in Fig. 1. As is expected, the character of the regions where the repulsive forces are found is that the one of the two slab permittivities is larger than the value for vacuum, whereas the other is smaller, and moreover, the greater the difference between two permittivities, the more easily the repulsive force is obtained. But in practical, it is hard to find real dielectrics with approximately non-dispersive permittivity smaller than unity over a sufficiently wide frequency range, and the Casimir forces are generally attractive between real dispersive ordinary dielectric slabs.

It is possible to control the sign of the Casimir force when the metamaterial with controllable permittivity and permeability described by Eq. (2) is introduced. Consider that slab A is the dispersive ordinary dielectric material and slab B is metamaterial, then we present

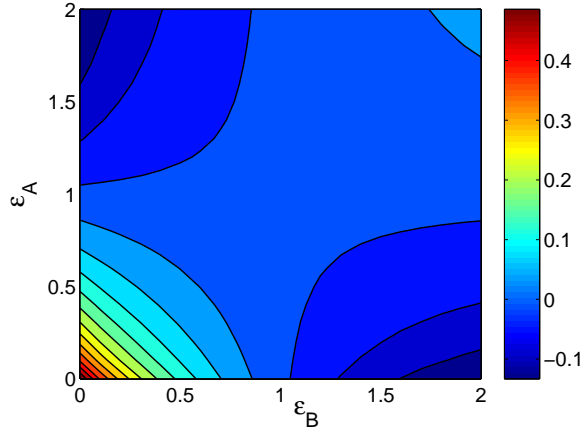


FIG. 1: Contour plot of F_r between two semi-infinite non-dispersive ordinary dielectric slabs as a function of the permittivities ϵ_A and ϵ_B (slab separation $a = \lambda_0/4$).

the influence of the characteristic frequencies of the metamaterial under the condition of the parameters of the ordinary dielectric being fixed. Fig. 2(a) illustrates the dependence of the relative Casimir force on the plasma frequencies of the metamaterial. The repulsive force tends to appear in the region where ω_{PeB} is decreased and ω_{PmB} is simultaneously increased. This behavior can be explained as follows. For slab A, most values of the impedance over the whole frequency range are no larger than unity, that is, the permittivity is no smaller than the permeability. Therefore, in order to satisfy the condition of formation of the repulsive force, slab B must have the permeability that exceeds the permittivity. We emphasize that repulsive force can be obtained if Z_A and Z_B are, respectively, smaller and larger than unity and the difference between them is great. The decreasing ω_{PeB} and the increasing ω_{PmB} correspond to the decreasing permittivity and the increasing permeability, and accordingly the repulsive force tends to appear. Fig. 2(b) shows the influence of the resonant frequencies of the metamaterial. By simultaneously increasing ω_{TeB} and decreasing ω_{TmB} , the attractive force can be changed to the repulsive force, which may be explained through a similar analysis.

It makes the sign control more freely when both the slabs are the metamaterials. Then we consider the influences of the characteristic frequencies of one metamaterial (slab B) when the parameters of the other metamaterial (slab A) are fixed, which are shown in Fig. 3. In order to distinguish from the case of dispersive ordinary dielectric slab, we let slab B have the magnetic properties stronger than the electric properties. As the parameters of slab B

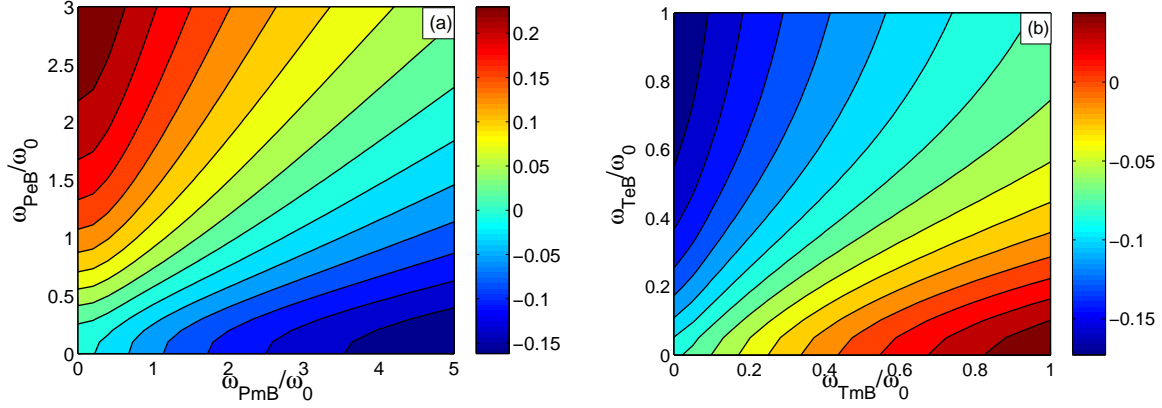


FIG. 2: Contour plot of F_r between the ordinary dielectric and metamaterial semi-infinite slabs as a function of the plasma and the resonant frequencies of the metamaterial [slab separation $a = \lambda_0/4$; characteristic frequencies: $\omega_{PeA} = \omega_0$, $\omega_{TeA} = 0$, $\gamma_{eA} = 10^{-2}\omega_{TeA}$, $\gamma_{\nu B} = 10^{-2}\omega_{T\nu B}$ ($\nu = e, m$), (a): $\omega_{TeB} = \omega_{TmB} = 0.5\omega_0$, (b): $\omega_{PeB} = 0.5\omega_0$, $\omega_{PmB} = 3\omega_0$].

chosen in the figures indicate, the magnetic resonant frequency is lower than the electric one, and the magnetic plasma frequency higher than the electric one, thus the impedances are in general larger than the impedance of vacuum. The appearance and gradually increase in magnitude of repulsive forces correspond to simultaneously stronger electric properties and weaker magnetic properties, which is opposite to the cases shown in Fig. 2. This confirms again the condition of the formation of the repulsive Casimir force that is stated above.

We proceed now to the dependence of the force on the distance between the slabs. The existence of the boundary surface in vacuum changes the zero-point energy, thus the mode densities are redistributed. There are more modes for certain frequencies between the macroscopic slabs than in the space outside the slabs, whereas there are fewer for other frequencies [24]. Consequently the force resulted from the inside and outside pressure difference can be attractive or repulsive. When the slab separation is adiabatically changed, the redistribution of the mode densities are correspondingly influenced, and then the Casimir force may be shifted to opposite direction. One can adjust the characteristic frequencies of the metamaterials to satisfy the condition of formation of repulsive forces that has been stated above at certain slab separations. In general, the Casimir force is attractive at very short distances, and as the slabs get further away from each other, the attractive force is gradually lowered and may become repulsive at certain distance, as is seen from Fig. 4. The repulsive force

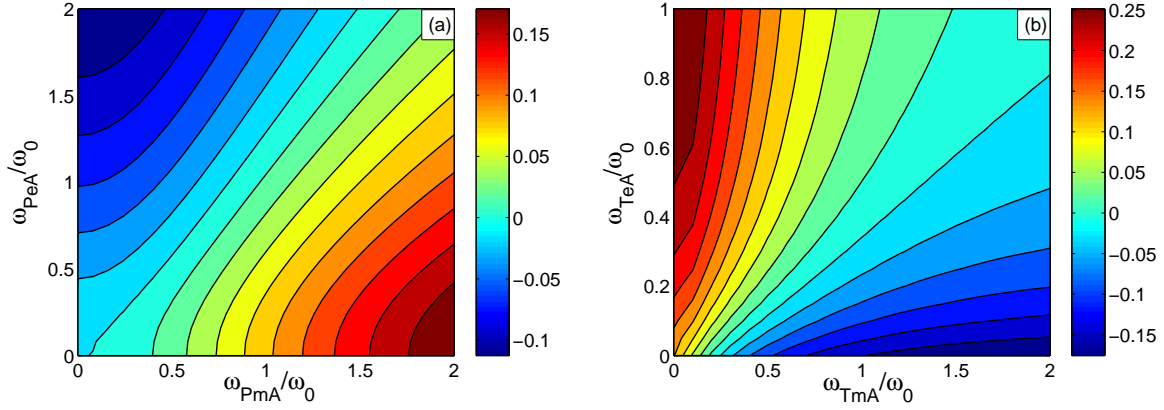


FIG. 3: Contour plot of F_r between two metamaterial semi-infinite slabs as a function of the plasma frequencies ω_{PeA} and ω_{PmA} [slab separation $a = \lambda_0/4$; characteristic frequencies: $\omega_{TeB} = 0.7\omega_0$, $\omega_{TmB} = 0.5\omega_0$, $\omega_{PeB} = 0.2\omega_0$, $\omega_{PmB} = 1.5\omega_0$, $\gamma_{\nu\zeta} = 10^{-2}\omega_{T\nu\zeta}$ ($\nu = e, m$, $\zeta = A, B$)], (a): $\omega_{TeA} = 0.5\omega_0$, $\omega_{TmA} = \omega_0$, (b): $\omega_{PeA} = \omega_{PmA} = \omega_0$.

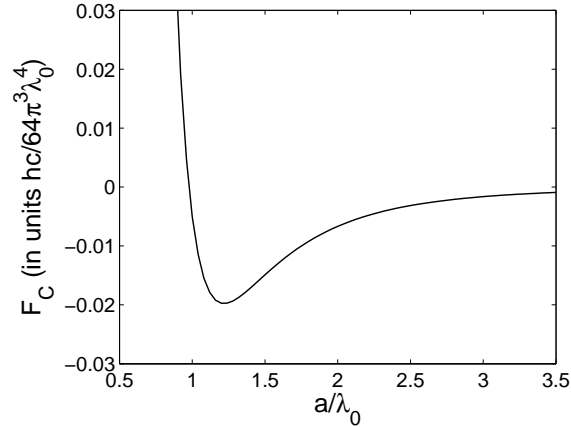


FIG. 4: Casimir force F_C between two metamaterial semi-infinite slabs as a function of the slab separation a [characteristic frequencies: $\omega_{PeA} = \omega_0$, $\omega_{TeA} = 0.5\omega_0$, $\omega_{PmA} = \omega_{TmA} = \omega_0$, $\omega_{PeB} = 0.2\omega_0$, $\omega_{TeB} = 0.7\omega_0$, $\omega_{PmB} = 1.5\omega_0$, $\omega_{TmB} = 0.5\omega_0$, $\gamma_{\nu\zeta} = 10^{-2}\omega_{T\nu\zeta}$ ($\nu = e, m$, $\zeta = A, B$)].

begins to grow larger and attains a maximum magnitude value, and then decreases again with the increasing slab separation.

An opposite situation can be found if one of the slabs is taken to be perfect conductor, which has the infinite permittivity $\epsilon \rightarrow \infty$ and accordingly the properties of perfect reflections $r_{TE} = -1$ and $r_{TM} = 1$. Through the tuning of the parameters of the other slab (metamaterial), one can obtain the reversion of the Casimir force sign change. As Fig. 5(a)

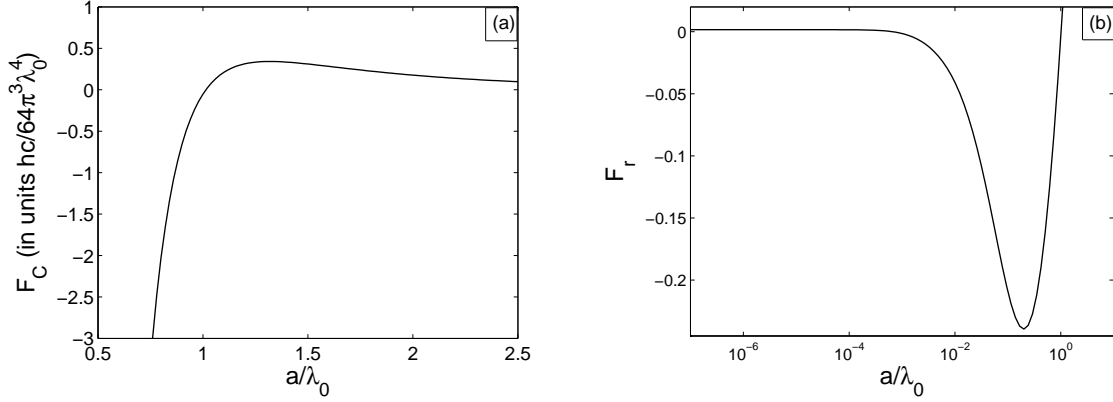


FIG. 5: Casimir force between a perfectly conducting semi-infinite slab and a metamaterial semi-infinite slab as a function of the slab separation. (a) and (b) show the dependence within different distance regions. [characteristic frequencies of the metamaterial: $\omega_{PeB} = 0.5\omega_0$, $\omega_{TeB} = 10^{-3}\omega_0$, $\omega_{PmB} = 3\omega_0$, $\omega_{TmB} = 0.7\omega_0$, $\gamma_{\nu B} = 10^{-2}\omega_{T\nu B}$ ($\nu = e, m$)].

indicated, the Casimir force between the perfectly conducting and metamaterial slabs may change from repulsion to attraction with the increasing slab separation. The existence of this type of restoring force presents the possibilities of the quasi-harmonic oscillation for the mechanical system in vacuum, and otherwise it may stabilize the system. Then we investigate the formation of this restoring force.

When the distances are large, the corresponding effective frequency range that has main contribution to the Casimir force integral is narrowed down to low frequencies. For the case of two metamaterial semi-infinite slabs as shown in Fig. 4, the static electromagnetic properties are much different between two slabs (cf. Fig. 6). The static impedances of two metamaterials are evaluated as: $Z_A(0) = \sqrt{\mu_A(0)/\epsilon_A(0)} \simeq 0.63$, $Z_B(0) = \sqrt{\mu_B(0)/\epsilon_B(0)} \simeq 3.04$. Thus in the large separation region the force appears to be repulsive. For the case of a perfectly conducting slab and a metamaterial slab as shown in Fig. 5(a), the chosen metamaterial slab possesses the static electric properties that exceed the static magnetic properties (cf. Fig. 7), which is seen from its static impedances $Z_B(0) = \sqrt{\mu_B(0)/\epsilon_B(0)} \rightarrow 0$. The perfectly conducting slab, whose impedance $Z \rightarrow 0$ at any frequencies, has the similar electromagnetic properties as the chosen metamaterial slab within low frequency range, therefore the force between them is attractive at large distances. When the separation of two slabs is relatively small, the corresponding effective frequency range is rather wide. For

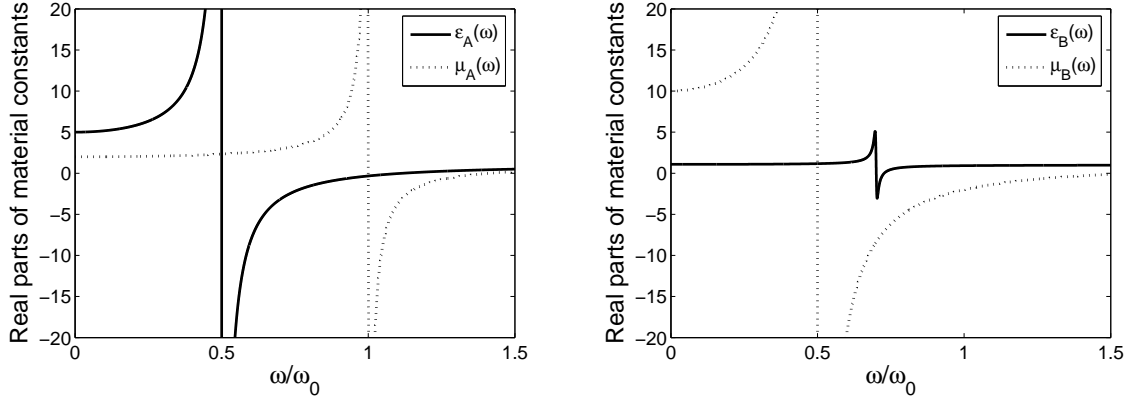


FIG. 6: Real part of the ϵ and μ of two metamaterials A and B chosen in Fig. 4 versus frequency.

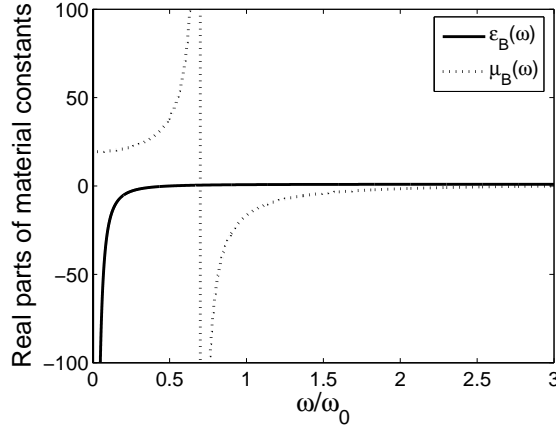


FIG. 7: Real part of the ϵ and μ of the metamaterial chosen in Fig. 5 versus frequency.

the metamaterial slab, although the material constants are dispersive, the total effect may lead to that the magnetic properties are stronger than the electric properties, or the other way round. Here the three examples of the metamaterials (slabs A and B in Fig. 6 and slab B in Fig. 7) may all belong to the former situation, thus at short distances, the attraction results from the similarity of the slabs, and the great difference between the properties of the perfectly conductor and the metamaterial gives rise to a repulsive force. The restoring force requests that there is repulsion at short distances and attraction at large distances, so in summary there is restoring force between the perfectly conductor and the metamaterial slabs, whereas the restoring force does not appear in the case of two metamaterial slabs.

One may adjust the values of the characteristic frequencies in order to obtain the large-separation attractive force between two metamaterial slabs. However, by doing so, the force

for the relatively short separation may generally be attractive as well, accordingly there is yet no restoring force between such metamaterial slabs. We attribute the formation of the restoring force to the speciality of the perfect conductor. The perfect conductor has the extremely strong electric properties at any frequencies. This extremeness makes the force sign change rather sensitive with respect to the variation of the properties of the other slab. The Casimir force for the above case within wider distance range is plotted in Fig. 5(b). In fact, the force is attractive at much shorter distances where the corresponding effective frequency ranges are quite wider. Thus for this case, the sign of the casimir force has changed twice with the increasing slab separation. The restoring force is only possible under such situation. The forces between perfectly conducting and metamaterial slabs at extremely short distances may either be repulsive, but then the properties of the chosen metamaterial, which give rise to repulsion at those short distances, generally also lead to repulsion at larger distances, and there is no restoring force at any distances.

In conclusion, we have studied cases of different directions of the Casimir force between parallel slabs, including ordinary dielectrics, metamaterials and perfect conductor [25]. It is found that the repulsive Casimir force can be obtained when the electromagnetic properties of two slabs are greatly different. We particularly focus on the restoring Casimir force which is found between the perfect conductor and the metamaterial, and the dependence of Casimir force on the slab separation is investigated from the point of view of the slab electromagnetic properties within different frequency range.

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